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FACULTY WORKING
PAPER NO. 1128

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Abstract

   This paper is concerned with alternative exact and asymptotic tests of
non-nested regression models. The relations between some tests are
developed to indicate how exact test statistics may be calculated from com-
puted asymptotic test statistics. The necessary and sufficient conditions
for exact tests to achieve maximum power against local alternatives are
presented. As an illustration of these conditions, it is shown that exact
tests may be applicable even when instrumental variable estimation is used,
and that not all exact tests attain maximum local power. The conditions
under which tests based on artificial linear regressions are asymptotically
equivelant to a Cox test of the null model are also examined.

Keywords: Asymptotic equivalence, Cox test, finite sample distribution,
           maximum local power, non-nested regression models.

1. Introduction

In recent years substantial contributions have been made in developing tests for non-nested regression models; see e.g., Pesaran and Deaton (1978), Davidson and MacKinnon (1981), Fisher and McAleer (1981) and Dastoor (1983). However, the theoretical advances have not led to many empirical applications. The reticence on the part of the econometric profession to use the available tests may stem, in part, from two related factors. First, the relationship between exact and asymptotic tests is not always transparent, and it is sometimes unclear when exact tests may be applied. Since virtually all existing tests have been based upon the modified likelihood ratio of Cox (1961, 1962), they are valid only asymptotically and little is yet known of their small sample properties. As the actual and nominal type I errors for asymptotic tests may differ substantially when the sample size is small, whereas exact tests have known finite sample null distributions, it is important to examine the conditions under which exact tests apply. Second, different tests, whether exact or not, may be asymptotically equivalent under the null hypothesis but not under the alternative. Furthermore, they may have different powers against local alternatives and it is not known, in general, which exact tests maximize local power.

The purpose of this paper is to investigate the issues addressed above. The plan of the paper is as follows. Finite sample relations among alternative asymptotic tests are given in Section 2. The relations between some exact and asymptotic tests are developed in Section 3, and it is shown how exact test statistics may be calculated from computed asymptotic test statistics. Necessary and sufficient conditions for exact tests to achieve maximum power against local alternatives are presented in Section 4 in an attempt to determine an optimal class of tests procedures. An example is

given to show that exact tests are available even when instrumental variable estimation is used, and that not all exact tests achieve maximum local power. The conditions under which tests based on artificial linear regressions are asymptotically equivalent to a Cox test are also examined. Some concluding remarks are given in Section 5.

2. Finite Sample Relations Between Alternative Tests

It is desired to test two non-nested linear regression models, H_0 and H_1, against each other, where the models are given by

$$H_0: \quad y = X\beta + u_0, \quad u_0 \sim N(0, I_n \sigma_0^2), \tag{1}$$

$$H_1: \quad y = Z\gamma + u_1, \quad u_1 \sim N(0, I_n \sigma_1^2). \tag{2}$$

In equations (1) and (2), y is an $n \times 1$ vector of observations on the dependent variable, X and Z are $n \times k$ and $n \times g$ matrices of n observations on k and g linearly independent regressors, β and γ are $k \times 1$ and $g \times 1$ vectors of unknown parameters, and u_0 and u_1 are $n \times 1$ vectors of disturbances. It is assumed that the matrices $n^{-1}X'X$ and $n^{-1}Z'Z$ converge to well-defined finite positive definite limits, and $n^{-1}X'Z$ is non-zero and also converges to a non-zero finite matrix. The two models are non-nested in the sense that at least one column of each regressor matrix cannot be expressed as a linear combination of the columns of the other. The design matrices X and Z are assumed to be fixed in repeated samples, although this assumption will be relaxed in Section 4.

Denoting $\theta_0' = (\beta', \sigma_0^2)$ and $\theta_1' = (\gamma', \sigma_1^2)$, the Cox test of H_0 is based upon the statistic

$$T_0 = (\hat{\ell}_0 - \hat{\ell}_1) = n[\text{plim}_0 \, n^{-1}(\hat{\ell}_0 - \hat{\ell}_1)]_{\theta_0 = \hat{\theta}_0}, \tag{3}$$

in which

$$\hat{\ell}_i = -1/2 \, n \log 2\pi - 1/2 \, n \log \hat{\sigma}_i^2 - 1/2 \, n \quad (i = 0,1)$$

is the maximized log-likelihood function under H_i, $plim_0$ denotes probability limit under H_0 and the second term in (3) is evaluated at $\theta_0 = \hat{\theta}_0$. The circumflex denotes maximum likelihood estimate, in which case $\hat{\sigma}_0^2 = n^{-1}y'(I-P)y$, $P = X(X'X)^{-1}X'$, $\hat{\sigma}_1^2 = n^{-1}y'(I-Q)y$ and $Q = Z(Z'Z)^{-1}Z'$. The Atkinson (1970) variation of the Cox test of H_0 is based upon

$$TA_0 = (\hat{\ell}_0 - \hat{\ell}_{10}) - n[plim_0 n^{-1}(\hat{\ell}_0 - \hat{\ell}_{10})]_{\theta_0 = \hat{\theta}_0}, \qquad (4)$$

in which

$$\hat{\ell}_{10} = -1/2\, n \log 2\pi - 1/2\, n \log \hat{\sigma}_{10}^2 - 1/2\, n(n^{-1}y'(I-QP)'(I-QP)y)/\hat{\sigma}_{10}^2,$$

and $\hat{\sigma}_{10}^2 = \hat{\sigma}_0^2 + n^{-1}y'P(I-Q)Py$ is a consistent estimate of $plim_0\, \sigma_1^2$ under H_0, while the expecatation of QPy equals the expectation of Qy under H_0. The second terms in (3) and (4) are asymptotically equivalent under H_0. Since, in addition, $plim_0\, \hat{\sigma}_1^2 = plim_0\, \hat{\sigma}_{10}^2 = plim_0\, n^{-1}y'(I-QP)'(I-QP)y = \sigma_{10}^2$, it follows that T_0 and TA_0 are asymptotically equivalent (for further details, see Fisher and McAleer, 1981). The important point to note in the preceding discussion is that both the Cox and Atkinson procedures modify a likelihood ratio by subtracting its asymptotic expectation evaluated at consistent estimates of the parameters under the null model. Another possible modification is given by

$$T_0^* = (\hat{\ell}_0 - \hat{\ell}_1^*) - n[plim_0\, n^{-1}(\hat{\ell}_0 - \hat{\ell}_1^*)]_{\theta_0 = \hat{\theta}_0}, \qquad (5)$$

where $\hat{\ell}_1^*$ is the calculated value of the likelihood function under H_1 when θ_1' is replaced by $(\hat{\gamma}^{*'}, \hat{\sigma}_{10}^2)$. As will be discussed in Section 4 below, we consider estimates $\hat{\gamma}^*$ of γ wnich are linear in y.

The Cox principle of testing non-nested models was introduced to the econometric literature by Pesaran (1974) for testing linear regression models, and by Pesaran and Deaton (1978) for testing non-linear systems of equations. In this paper, we will refer to the test derived by these

authors as the Cox test. The test may be modified using Atkinson's suggestion of evaluating the entire statistic under the null hypothesis. Denoting a consistent estimate of the asymptotic variance of TA_0 as $\hat{V}_0$, the Atkinson test, namely $NA_0 = TA_0/(\hat{V}_0)^{1/2}$, is distributed approximately as $N(0,1)$ in large samples. Fisher and McAleer (1981) show that NA_0 may be written as

$$NA_0 = \frac{1/2\ n(\hat{\sigma}_1^2 - \hat{\sigma}_{10}^2) + 1/2\ y'(I-P)Q(I-P)y}{\sigma_0(y'PQ(I-P)QPy)^{1/2}},$$

which is equivalent to

$$NA_0 = -y'PQ(I-P)y/\{\hat{\sigma}_0(y'PQ(I-P)QPy)^{1/2}\}. \tag{6}$$

Apart from σ_0, NA_0 may be calculated from the artificial regression

$$y = X\beta + \alpha QPy + u. \tag{7}$$

The t-ratio for the ordinary least squares (OLS) estimate of α in (7), namely

$$t(\hat{\alpha}) = y'PQ(I-P)y/\{\hat{\sigma}(y'PQ(I-P)QPy)^{1/2}\},$$

in which $\hat{\sigma}^2 = (n-k-1)^{-1}\{y'(I-P)y - (y'PQ(I-P)y)^2/(y'PQ(I-P)QPy)\}$ is the estimated error variance from (7), is asymptotically equivalent to NA_0 under H_0. This result follows from the fact that $\text{plim}_0\ \hat{\sigma}^2 = \text{plim}_0\ \hat{\sigma}_0^2 = \sigma_0^2$. In the remainder of this section and the next, we will concentrate upon asymptotic approximations to the Cox test based upon artificial regressions such as the one given in (7).

Three simple asymptotic procedures have been developed by Davidson and MacKinnon (1981) for testing non-nested models. These tests are an approximation to the Cox test and are also very simple to calculate. In the linear case, the large sample J and P tests are identical, while both are preferred to the C test. We shall provide below the finite sample relations between the J and C tests. The sample means of y under H_0 and

H_1 are given by $Py = X\hat{\beta}$ and $Qy = Z\hat{\gamma}$, respectively, and the J test is the t-ratio for the OLS estimate of α in the artificial regression

$$y = (1 - \alpha)X\beta + \alpha Qy + u. \tag{8}$$

A two-tailed test of $\alpha = 0$ in (8) is a test of H_0, whereas a test of H_1 is performed by testing $\lambda = 0$ in the artificial regression

$$y = (1 - \lambda)Z\gamma + \lambda Py + u. \tag{9}$$

If Py is substituted for $X\beta$ in (8), we have the C test of H_0, namely the test of $\alpha = 0$ in

$$y = (1 - \alpha)Py + \alpha Qy + u, \text{ or } (I - P)y = \alpha(Q - P)y + u. \tag{10}$$

Davidson and MacKinnon (1981, p. 783) recommend the C test as a preliminary test of H_0 because <u>asymptotically</u> it "...has variance less than unity under H_0." The fact that the C test is undersized (i.e., the actual size of the test is less than its nominal size) when both α and the error variance are estimated by maximum likelihood methods is demonstrated below. Denoting the large sample tests of $\alpha = 0$ (i.e., the t-ratios based upon maximum likelihood estimation) from (8) and (10) as $\bar{t}(\hat{\alpha}_J)$ and $\bar{t}(\hat{\alpha}_C)$, respectively, it follows that

$$\bar{t}^2(\hat{\alpha}_J) = (y'Q(I - P)y)^2/(\hat{\sigma}_J^2(y'Q(I - P)Qy)),$$

$$\bar{t}^2(\hat{\alpha}_C) = (y'Q(I - P)y)^2/(\hat{\sigma}_C^2(y'(Q - P)^2 y)),$$

where $\hat{\sigma}_J^2 = n^{-1}\{y'(I - P)y - (y'Q(I - P)y)^2/(y'Q(I - P)Qy\}$

and $\hat{\sigma}_C^2 = n^{-1}\{y'(I - P)y - (y'Q(I - P)y)^2/(y'(Q - P)^2 y)\}$ are the maximum likelihood estimates of the error variances from (8) and (10), respectively. Since $y'(Q - P)^2 y - y'Q(I - P)Qy = y'(I - Q)P(I - Q)y$ is strictly positive (even asymptotically) when H_1 is not fitting perfectly, it follows that $\hat{\sigma}_C^2 > \hat{\sigma}_J^2$, and hence

$$\bar{t}^2(\hat{\alpha}_J) > \bar{t}^2(\hat{\alpha}_C). \tag{11}$$

Since the J test has correct size asymptotically, the C test must be under-sized.

It should be noted that there are two inherent problems with the C test. First, the formula for the variance of $\hat{\alpha}_C$ is incorrect. Second, the estimated error variance used in calculating the t-ratio for $\hat{\alpha}_C$ is inappropriate for finite samples. When the two tests are calculated from a least squares package, the denominators of $\hat{\sigma}_J^2$ and $\hat{\sigma}_C^2$ become $(n-k-1)$ and $(n-1)$, respectively, so that the J and C tests are given by $t(\hat{\alpha}_J)$ and $t(\hat{\alpha}_C)$. The relationships between the tests based upon maximum likelihood and least squares procedures are $n\, t^2(\hat{\alpha}_J) = (n-k-1)\bar{t}^2(\hat{\alpha}_J)$ and $n\, t^2(\hat{\alpha}_C) = (n-1)\bar{t}^2(\hat{\alpha}_C)$, so that the inequality between the J and C tests may be written as

$$(n-1)\, t^2(\hat{\alpha}_J) > (n-k-1)\, t^2(\hat{\alpha}_C). \tag{12}$$

Strictly speaking, there should be $(n-k-1)$ degrees of freedom for $t(\hat{\alpha}_C)$, as well as for $t(\hat{\alpha}_J)$, since k degrees of freedom are lost in estimating β to obtain (10) from (8). The inequality given by (12) refers solely to the t-ratios obtained from OLS estimation of (8) and (10). In spite of inequality (11), when k is large relative to n and OLS estimation is used, it is clear from (12) that $t(\hat{\alpha}_J)$ may be exceeded by $t(\hat{\alpha}_C)$.

Under H_0, Py is distributed independently of $(I - P)y$, the vector of residuals obtained from least squares estimation of (1). It is noteworthy that Qy is _not_ distributed independently of $(I - P)y$. Therefore, the J test is not an exact test for H_0 when there is more than one column in Z that is linearly independent of the columns of X. However, when Z contains only one column that is linearly independent of those of X (e.g., when there is one regressor in H_1 that is not in H_0), the J test of H_0 does have the exact t distribution in small samples. Following Atkinson (1970),

Fisher and McAleer (1981, p. 109) evaluate the entire statistic under H_0 by replacing Qy in (8) with QPy, as in (7), leading to

$$y = (1 - \alpha)X\beta + \alpha QPy + u, \tag{13}$$

in which a two-tailed test of $\alpha = 0$ is a test of H_0.

Milliken and Graybill (1970) have shown that if one or more functions of the linear predictions of the linear null model, namely Py, are added to that model, the test statistic of the significance of the additional regressors will have the exact F distribution when the disturbances are normal. In (13), the single additional regressor, QPy, is a function of Py and hence is distributed independently of $(I - P)y$. Thus, the JA test, which is the t-ratio for the OLS estimate of α, has the exact t distribution with (n-k-1) degrees of freedom under H_0. If the true spirit of the Cox principle is to be observed, however, the roles of the null and alternative models should be reversed. The JA test of H_1, which is the t-ratio for the OLS estimate of λ in

$$y = (1 - \lambda)Z\gamma + \lambda PQy + u, \tag{14}$$

has the t-distribution with (n-g-1) degrees of freedom under H_1.

The general results of Milliken and Graybill have been used directly by McAleer (1983) to test a linear null model against several non-nested, non-linear alternatives simultaneously. The resulting test statistic has a central F-distribution under the null but not a non-central F-distribution when the null is false. There is presently very little known about the power of the Milliken and Graybill procedure, and indeed the test may not even be unbiased. Pesaran (1982b) has shown that the JA and J tests have equal power in large samples when H_0 is tested against <u>local</u> alternatives and $k \geq g$. However, little is known of power considerations in the interesting case of <u>fixed</u> alternatives, or when $k < g$. Of course, it is

possible for k to exceed g through underspecifying the alternative, in which case the tests of the null may not even be consistent. For more on this, see McAleer et al. (1982), where it is also shown that overspecification does not affect the consistency of the tests.

3. Relations Between Some Exact and Asymptotic Tests

The numerator of the JA statistic from (13) is given by

$$\hat{\alpha}_{JA} = (y'PQ(I - P)y)/(y'PQ(I - P)QPy), \tag{15}$$

and the squared t-ratio from OLS estimation of $\hat{\alpha}_{JA}$ is

$$t^2(\hat{\alpha}_{JA}) = (y'PQ(I - P)y)^2/(\hat{\sigma}^2_{JA}(y'PQ(I - P)QPy)), \tag{16}$$

where $\hat{\sigma}^2_{JA} = (n-k-1)^{-1}\{(y'(I - P)y) - (y'PQ(I - P)y)^2/(y'PQ(I - P)QPy\}$ is the OLS estimate of the error variance. The exactness of the JA test relies upon the independence of QPy and $(I - P)y$ under H_0 so that, given QPy, $E_0(y'PQ(I - P)y) = 0$ and $E_0(\hat{\sigma}^2_{JA}) = \sigma^2_0$, where E_0 denotes expected value under H_0. Thus, $\hat{\sigma}^2_{JA}$ is an unbiased estimator of σ^2_0 and $\hat{\alpha}_{JA}$ has zero expectation under H_0.

Transforming both sides of (13) by $(I - P)$ annihilates X and yields the PA test of H_0, namely the test of $\alpha = 0$ in

$$(I - P)y = \alpha(I - P)QPy + u. \tag{17}$$

It should be noted that there are only (n-k) independent observations in (17) since the k elements of β have already been estimated. Therefore, the t-ratio for $\hat{\alpha}_{PA}$ from (17) must be multiplied by $(n-k-1)^{1/2}/(n-1)^{1/2}$ to obtain a test statistic which has the exact t distribution with (n-k-1) degrees of freedom. If Xβ is replaced by Py in (13), rearrangement yields

$$(I - P)y = -\alpha(I - Q)Py + u, \tag{18}$$

in which the test of $\alpha = 0$ is, say, the CA test of H_0. Note that transforming both sides of (18) by $(I - P)$ yields the PA test in (17).

It is of interest to obtain the OLS estimate of λ in (9) to test H_1, namely

$$\hat{\lambda}_J = (y'P(I - Q)y)/(y'P(I - Q)Py). \tag{19}$$

The numerator of $\hat{\lambda}_J$ involves the inner product of Py and $(I - Q)y$ so that the J test of H_1 does not have the exact t distribution, just as it is not exact for H_0. However, consider testing H_0 by testing the null hypothesis $\lambda = 1$, for which the squared t-ratio from OLS estimation of λ in (9) is

$$t^2(1 - \hat{\lambda}_J) = (y'PQ(I - P)y)^2/(\hat{\sigma}^2_{J1}(y'P(I - Q)Py)), \tag{20}$$

in which $\hat{\sigma}^2_{J1} = (n-g-1)^{-1}\{(y'(I - Q)y) - (y'P(I - Q)y)^2/(y'P(I - Q)Py\}$. Note that, apart from the estimated error variances, the expression for $t^2(1 - \hat{\lambda}_J)$ in (20) is identical to that for $t^2(\hat{\alpha}_{CA})$ from (18), in spite of the fact that the non-nested models tested in each case are different. Thus, if the correct formula (in so far as testing H_0) for the variance of $1 - \hat{\lambda}_J$ were used, together with $\hat{\sigma}^2_{JA}$ as the estimated error variance, it is possible for a test of H_1 based upon (9) to be exact. Although it would be a straightforward matter to adjust (20) by the factor

$$d = (\hat{\sigma}^2_{J1}(y'P(I - Q)Py))/(\hat{\sigma}^2_{JA}(y'PQ(I - P)QPy))$$

to obtain an exact test for H_0 based upon the J test for H_1, it is clearly more straightforward to calculate the JA statistic for H_0 directly from (13). It is interesting to note that simple adjustments for the variance and the estimated error variance can lead to an exact J test of a model that was originally designated as the alternative. Since exactness of a test arises from independence of the additional stochastic regressor and the residual vector of the tested model, a test that is exact for the null cannot be exact for the alternative. Thus, while the JA test of

$\lambda = 0$ in (14) is exact for H_1, it cannot be modified to give an exact test of H_0 by any transformation similar to d.

Since $\text{plim}_0 d = (\beta'(\text{plim } n^{-1}X'(I - Q)X)\beta)/(\beta'(\text{plim } n^{-1}X'Q(I - P)QX)\beta) > 1$, where plim_0 denotes probability limit under H_0, it follows that the CA test and the J test of H_0, when set up initially to test H_1, will not reject H_0 frequently enough, even though both statistics have zero means under H_0. The reason for this is that the variances of both statistics are incorrect for testing H_0, in spite of the fact that $\hat{\sigma}_{CA}^2$ and $\hat{\sigma}_{J1}^2$ (an estimated error variance from the J test of H_1) are consistent for the 'true' error variance under H_0.

4. Exact Tests With Maximum Local Power

The exact nature of various tests refers to the probability of committing a type I error only. It is clear that substituting any estimator of the form $\tilde{\gamma} = VPy$ for γ in

$$y = (1 - \alpha)X\beta + \alpha Z\gamma + u, \tag{21}$$

where V is a fixed g x n matrix of rank g, will lead to an exact test of H_0 since ZVPy is distributed independently of $(I - P)y$. However, an analysis of the properties of alternative tests would not be complete without a discussion of their relative powers. For cases in which the dimension of the tested model is not exceeded by that of the alternative, Pesaran (1982a) has examined the local power of tests of non-nested models. The conditions under which a test of H_0 based on an artificial linear regression such as (21) has maximum power against local alternatives, in addition to being consistent and asymptotically distributed as unit normal, are stated by Pesaran (1982b). The conditions are presented below for exact tests, assuming that $k \geq g$:

CONDITION (C1): $\lim_i (VPu_i) = 0$ under $H_i (i = 0,1)$.

CONDITION (C2): $\lim (VX) = D_0$, where D_0 is finite and non-zero.

CONDITION (C3): $\lim (VPZ) = D_1$, where D_1 is a positive definite matrix.

CONDITION (C4): under local alternatives, $b_1 \lim (VPZ)\gamma = b_2\gamma$, in which b_1 and b_2 are constants.

Conditions (C1) - (C3) are essentially a restatement of those given in Pesaran (1982b), with due consideration for exact tests. The four conditions stated are necessary and sufficient for an exact test to be consistent, asymptotically $N(0,1)$ under H_0, and to maximize local power. Condition (C4) arises as a consequence of the Cauchy-Schwarz inequality (see e.g., Rao, 1973, p. 54), and effectively requires that the limit of VPZ be proportional to an identity matrix.

It should be noted that there are many matrices V which will satisfy the four conditions stated above. However, by relating the test statistic based upon the artificial regression in (21) to a Cox-type test based on equation (5) in a systematic manner, we can restrict the possibilities for V considerably. This may be shown as follows. On the basis of (5), the likelihood function for H_1 evaluated at $(\hat{\gamma}^{*\prime}, \hat{\sigma}^2_{10})$, where $\hat{\gamma}^* = VPy$, is given by

$$\hat{\ell}_1^* = - 1/2\, n \log 2\pi - 1/2\, n \log \hat{\sigma}^2_{10} - 1/2\, n\, (n^{-1}y'(I-ZVP)'(I-ZVP)y)/\hat{\sigma}^2_{10}.$$

Since it is required that $\text{plim}_0\, n^{-1}y'(I - ZVP)'(I - ZVP)y = \sigma^2_{10}$, which is equivalent to requiring $\lim n^{-1}X'(I-ZVP)'(I-ZVP)X = \lim n^{-1}X'(I-QP)'(I-QP)X = \lim n^{-1} X'(I-Q)X$, two possible solutions are $ZV = Q$ and $ZV = QP$. However, both of these possibilities simply lead to the JA test considered previously, and which is known to be asymptotically equivalent to a Cox test. Thus, requiring tests based upon artificial regressions such as (21) to be asymptotically equivalent to a Cox test reduces the possibilities

for the matrix V. In general, it rules out several forms considered by Pesaran (1982b), such as $ZV = QPQ$.

In the remainder of this section we will assume that only the four conditions stated hold, with X and Z being stochastic. It is easy to show that exact tests are available for H_0 even when Z is independent of u_0 but not of u_1. In this case, a set of instruments given by the matrix W of rank h ($\geq$ g) may be used for consistent estimation under H_1. If Z is not independent of u_1, the JA test of H_0 will be exact regardless of whether OLS or instrumental variable (IV) estimation is used for H_1. This result holds because Z is assumed to be independent of u_0, even though it is not independent of u_1.

In the preceding discussion, we have not assumed that all the columns of X and Z are linearly independent of each other. Therefore, condition (C1) may not be satisfied under H_1, although conditions (C2) – (C4) are satisfied. For instance, let X and Z be partitioned as $[X_1 : Z_2]$ and $[Z_1 : Z_2]$, respectively, where X_1 and Z_1 are linearly independent. If neither Z_1 nor Z_2 is independent of u_1, condition (C1) does not hold. However, if Z_2 is independent of u_1, (C1) is satisfied and hence the power of the JA test of H_0 is maximized whether OLS or IV estimation is used for H_1. Since V equals $(Z'Z)^{-1}Z'$ and $(Z'SZ)^{-1}Z'S$ for the JA test when OLS and IV estimation are used, respectively, in which $S = W(W'W)^{-1}W'$, then the limit of VPZ will be proportional to an identity matrix. Thus, the JA test of H_0 is not only exact, but in this case also attains maximum power against local alternatives whether OLS or IV estimation is used.

As a word of warning, however, it should be stressed that some exact tests may not attain maximum local power. The most general form for V is $V = (Z'S_1Z)^{-1}Z'S_2$, where S_1 and S_2 are both projection matrices derived

from sets of instruments. It is clear that setting γ in (21) to $(Z'S_1Z)^{-1}Z'S_2Py$ does not alter the exactness of the test of H_0. Special cases we have already examined are $S_1 = S_2 = I$ and $S_1 = S_2 \neq I$. Another possibility is when $S_1 \neq S_2$, in which case (C4) will not be satisfied unless $S_1X = S_2X = X$. Since it has not been assumed that $h \geq k$ or that the set of instruments for H_1 contains all the columns of X, it follows that an exact test does not necessarily maximize power against local alternatives.

5. Concluding Remarks and Future Research

In this paper we have established finite sample relations among some exact and asymptotic tests of non-nested regression models. Necessary and sufficient conditions have been provided for exact tests to attain maximum local power. It was also shown that the JA test satisfies the above conditions and is also asymptotically equivalent to a Cox test. While Godfrey and Pesaran (1983) consider the finite sample size and power properties of various exact and asymptotic tests against well-specified alternatives, there are, however, still many issues that remain unanswered, and we mention some of those that are currently under investigation. First, little is known about the level of significance and power properties of the JA test when it is not distributed exactly. Second, although Dastoor and McAleer (1984) have obtained some analytical results on the relative powers of paired and joint tests of non-nested models, the small sample properties of these tests are as yet unknown. Third, the robustness of the tests has not been studied in the presence of non-standard error structures and specification errors of various types. There is also a need to develop simple test procedures for non-nested models under a general error specification such as non-normal and serially dependent errors. Finally, since it has been observed in Pesaran (1982a) that a Cox test rejects a 'true' null too

frequently, finite sample tests for normality of asymptotic test statistics would be useful.

Acknowledgement

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